The mereological structure of informational entities

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Abstract. This article provides the basis of a formal axiomatic system for a mereology of informational entities based on the idea of informational fillers that can occupy informational slots, such as the same word that can be used in different sentences. It is inspired by Karen Bennett's mereological system that enables a whole to have a part "twice over", but differs from it in several key points, such as the acceptance of empty slots, and the possibility for slots to have slots. Informational slots are analyzed as informational entities that can carry aboutness.

Keywords. Mereology, Information content entity, Informational structure

1. Introduction

Documents are a primary source of data. Consider the field of medicine: many data about which medications a patient takes (or is likely to take) are extracted from prescription documents written to him or her by doctors, or from drug dispensing reports written by pharmacists [1]. Forms and surveys are another important source of data.

As we will see below, documents receive growing attention in information systems and ontologies. However, the informational entities that compose such documents are often characterized simply as entities that are "about" something, and their analysis is too basic to enable an accurate and manageable representation of data and information in various fields. To address this problem, we will provide a mereological analysis of the structure of documents, as the meaning of a complex document depends on the meaning of its parts. Interestingly enough, this task will lead to a foundational challenge to reconsider classical mereological systems that are traditionally used in formal ontology. We will here only consider documents that are filled with natural language, excluding pictures, musical partitions, etc. – although this limitation could be lifted.

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2. Preliminaries and state-of-the-art works

2.1. Mereology among universals or particulars

Mereology is a formal study of the part-whole relation, which constitutes a mainstay of ontological practice. We may consider building a mereology at two levels: between particulars, or between universals. This article will focus on a mereology of particulars of informational entities. Former work by Masolo and Vieu [2] has focused on a mereology of universals (not necessarily those of informational entities). Particulars of informational entities and universals of non-informational entities share some commonalities [3, p. 105-107], such as the ability to be multiply localized. This is reflected in the fact that Bennett's [4] mereological work, used for building a mereology of universals in the work by Masolo and Vieu [2], will also serve as a basis for our formal ontology of informational entity particulars.

2.2. Classical extensional mereology

Different mereological systems have been proposed. The most standard is sometimes informally called "Classical Extensional Mereology" (CEM) (not necessarily in Simon's [5] sense). CEM embraces "ground mereology" [6] according to which parthood is a (partial) ordering relation (reflexive, antisymmetric, and transitive) and that accepts the two following principles (the former being entailed by the latter):

- Weak Supplementation Principle (WSP): If *x* is a proper part of *y*, then there is some *z* such that *z* is a part of *y* and *z* is disjoint from (i.e. does not overlap) *x*.
- Strong Supplementation Principle (SSP): If *y* is not part of *x*, then there is some *z* such that *z* is a part of *y* and *z* is disjoint from *x*.

Note that SSP implies that two different entities cannot have exactly the same proper parts. For instance, the upper ontology DOLCE [7] builds upon "general extensional mereology" [6] satisfying both WSP and SSP; whereas, another upper ontology Basic Formal Ontology (BFO 2.0) [3] adopts a "minimal extensional mereology" [5] including WSP but not SSP. Although Simons [5] endorses WSP and recommends dropping out SSP, WSP is still controversial [8].

Most importantly, CEM is committed to the principle that for every composite *x*, *x* cannot have *y* as a part many times over [4][9]. We will reject below this principle because it turns out to be unsuitable for developing a mereological account of informational entities.

2.3. Informational entities in conceptual modeling and ontology

There are some existing works on informational entities in the domain of conceptual modeling and ontology. For instance, the Functional Requirements for Authority Data (FRAD) [10] introduces an entity named "expression": "The intellectual or artistic realization of a work in the form of alphanumeric, musical, or choreographic notation, sound, image, object, movement, etc., or any combination of such forms." To take another example, the Unified Nations System Document Ontology (UNDO) [11] aims to provide a framework for the formal description of all entities and the relations that hold among them in the documents of the United Nations. Both the FRAD and the

UNDO are largely practically motivated and they leave room for meticulous ontological analysis of documents and information. Finally, CIDOC [12] is a lightweight ontology aimed at cultural heritage domain. It includes a mereological relation for information objects (incorporates), but its axiomatization is very limited.

Two ontologies based on BFO deal with informational entities. First, the Information Artifact Ontology (IAO) [13] introduces the class *Information Content Entity* (ICE), the instances of which can be documents, databases, and digital images, as a subclass of *Generically Dependent Continuant* (whose instances, intuitively, “can migrate from one bearer to another through a process of copying” [6] [p. 179]). ICEs are *about* some “portion of reality” which encompass all the BFO particular entities (including other ICEs) but also universals, relations, and “configurations” (e.g., the cat being on the mat). Second, the Information Entity Ontology in the Common Core Ontologies (CCO) [14] identifies three subkinds of the IAO:is_about relation: describes (used for e.g., reports and representations), prescribes (used for e.g., plans and artifact specifications), and designates (used for e.g., names and other identifiers).

Finally, Masolo et al. [15] notion of description is based on DOLCE, and characterized as follows: “different expressions (…) can be associated to the same description” (generic dependence) and “descriptions must be encoded on (…) physical supports” (concretization). This notion has been exploited within ontology patterns for describing information objects [16], but a full-fledged formal ontology of informational entities has not been pursued within DOLCE yet.

3. A classification of informational entities

3.1. A brief consideration on informational entities, aboutness, and semiotics

Documents have a dual face: physical objects on one hand (e.g. a copy of the book *Labyrinths*), and informational content that can be concretized by such physical objects on the other hand (e.g. the informational content shared by all such copies of *Labyrinths*). We will here be interested in this second sense of documents.

Many documents are constituted by sentences, which can be decomposed in words. Words written in alphabetic systems can be decomposed in letters. All those entities will be called here ‘Informational entities’ (IEs). The models of informational entities presented above seem to share the premise that informational entities are about (synonyms: “refers to”, “represents”, “mentions”) something. This is indeed often the case with documents, (declarative) sentences therein, and words that constitute the sentences: e.g., the word “cat” can be generally taken to be about the class *Cat*. It may not be the case of all informational entities, however. For instance, in most contexts, letters such as “A” are not about anything. Therefore, IEs should not be identified with ICEs from IAO, as they are not necessarily about something.

Aboutness remains a notoriously elusive notion, despite some recent philosophical works [17]. We will not delve into its nature (but see section 5.4), but will presuppose that aboutness emerges from a semiotic system. Relatively neglected in ontology [18], semiotics analyzes representations in terms of the triad of a sign, an object and an interpreter. That is to say, meaning is “an attribution of significance by some sign users for other sign users for some designated purposes” [18] [p. 120]. Therefore, for example, the letter “A” might be about something, but only when somebody refers to “A” with the intention of conveying some meaning (e.g., an excellent grade) to another person. To take seriously semiotic considerations in ontology may require a foundational investigation into language [18], which is outside the scope of this paper. What we will seek below is a mereological theory of informational entities.

3.2. The need for information slots

It seems that some informational entities can be found in several documents: for example, the IE 'flu' can be found in a string $dict_0$ in a medical dictionary that reads 'flu = an infectious disease caused by an influenza virus', and in a (here idealized) diagnosis $diag_0$ written by Dr. House about John Doe that reads: 'John Doe / flu / Dr. House'. To account for this phenomenon, we will introduce, following Bennett [4], the classes of Information Slot (IS) (somewhat akin to CCO's Information Structured Entity, although to our knowledge, the nature of the latter has not been investigated in detail yet) and Information Filler (IF), both being subclasses of the class of IE: the same individual IF 'flu' can be found in both $diag_0$ and $dict_0$, as filling different individual ISs.

Similarly, consider the chain of characters IE_1='ab' and IE_2='ba': we will consider that IE_1 has two ISs '1st letter$[]_1$' and '2nd letter$[]_1$', and that IE_2 has two ISs '1st letter$[]_2$' and '2nd letter$[]_2$'. The same individual filler 'a' occupies '1st letter$[]_1$' and '2nd letter$[]_2$', and the same individual filler 'b' occupies '1st letter$[]_2$' and '2nd letter$[]_1$'.

3.3. Information slots can be filled by information fillers

Suppose that in a hospital, all diagnostic documents have the following structure: 'patient[] condition[] doctor[]'. That is, any diagnostic report at this hospital has (at least) three ISs, each of which can be filled by an IF. When we write "s[x]", x refers to an IF and s to an IS that is filled by x.

We will also consider that IEs can have a structure even if this structure is not filled – that is, they can have a "mere mereological structure" (see section 5.6). This means that in our ontology, an IS does not need to be filled by an IE.

Note that two documents of the same type do not have the same ISs, although they can be filled with the same kinds of IFs. Suppose that Dr. House fills the document $diag_0$: 'patient['John Doe'$]_0$ condition['Flu'$]_0$ doctor['Dr. House'$]_0$' and $diag_1$: 'patient['Jane Brown'$]_1$ condition['Asthma'$]_1$ doctor['Dr. House'$]_1$'. Although $diag_0$ and $diag_1$ have the same structure, they have different ISs: 'patient$[]_0$' is different from 'patient$[]_1$', 'doctor$[]_0$' is different from 'doctor$[]_1$', etc. However, 'patient$[]_0$' and 'patient$[]_1$' are instances of the same class *IS for patient name*, 'doctor$[]_0$' and 'doctor$[]_1$' are instances of the same class *IS for doctor name*, etc. On top of that, the same particular IF 'Dr. House' fills both particular ISs 'doctor$[]_0$' and 'doctor$[]_1$'.

As we have seen earlier, not only words can occupy ISs: the same letter can appear in several words by occupying several different ISs. For example, in the word 'aa', the same particular IF 'a' occupies the IS '1st letter[]'$_{aa}$ and the IS '2nd letter[]'$_{aa}$.

3.4. IS as a GDC that can be concretized

To analyze ISs and IFs, we anchor them in the IAO ontological framework for ICEs in such a way that informational entities are generically dependent upon their bearers (see [19] for a detailed discussion on generic dependence) and exist by being "concretized". However, we extend this idea to all informational entities, including those that are not ICEs. The letter "A", for instance (even if not an ICE, since it is not about anything), may be concretized as an ink pattern on a paper, or as a pixel pattern on a computer screen.

Suppose there are two concretizations of $diag_0$: a first one printed on paper, and concretized by the ink pattern p_1, and a second one on my computer screen, concretized by a pixel pattern p_2. Let's call IF_1='John Doe', IF_2='Flu' and IF_3= 'Dr. House' the three IFs that constitute what we will call the "content" of $diag_0$. IF_1, IF_2 and IF_3 are concretized by (parts of) p_1, as well as (parts of) p_2. In our ontological framework, each IS in $diago_0$ is also concretized (at least) twice, since $diag_0$ is concretized twice (in p_1 and p_2). ISs are in this respect similar to IFs.

A difficulty with ISs lies in pinpointing their concretizations: it might be expected that diagnostic sheets at a hospital should be filled with the name of a doctor at the bottom part of the sheet, without anything indicating the need for such a name on the paper. Even if something indicates it, such as the words "Doctor name" written on the paper, those words are not an IS; rather, they are an IF that indicates the (otherwise invisible, but socially expected) existence of an IS. Maybe, if the prescription is concretized on a sheet of paper, such an IS would be concretized by a BFO:*Site* [3, pp. 112-113]. For example, if $diag_0$ is printed on a paper, the slot '$doctor[]_0$' in $diag_0$ is concretized by a site on the printed document that is occupied by the ink pattern on the paper concretizing the IF 'Dr. House'. Another possibility would be that ISs are concretized by some cognitive structure in a collective of agents, reflecting on the social nature of ISs. We will not elaborate further on this question in this paper, and instead focus on the axiomatic mereology of the system.

4. An axiomatization of mereological relations among informational entities

We will first present an axiomatical mereological system for IEs (4.1-4.4) inspired by Bennett's system [4], and then show in 4.5 how it differs from it.

4.1. Ground axiomatization

4.1.1. Key predicates

Let us adapt Bennett's mereology, where $P_S sy$ means that s is an IS (what Bennett calls a "slot") of y, Fxs means that x fills s, and Pxy means that x is a part of y. We introduce as primitive the unary predicate S = "being a slot", and the binary predicates F and P_S, where P_S is defined on the domain S:

(AX0) **Only slots are slots of something** $\quad P_S sx \rightarrow Ss$

We then define the following predicates FL = "being a filler" and HS = "having a slot":

(DEF0) **Have-slot and filler** $\quad HSz :=_{def} \exists s\ P_S sz$

$\quad FLx :=_{def} \exists s\ Fxs$

We will build a mereological theory on the domain of fillers and slots. This means that each non-slot entity under consideration is a filler, and therefore fills some slot: **$diag_0$**, for example, would fill a slot "$diagnosis[]_0$" that may not be a slot of any filler. This conception fits with the idea that there are no "free-floating" ICEs, but that they always appear in a context with some social expectations defined by the semiotic system on which they depend.

x is a *proper* part of a filler y if x fills a slot s of y:

(DEF1) **Proper filler-parthood** $PPxy :=_{def} FLy \;\&\; \exists s\, (P_S sy \;\&\; Fxs)$

Note that FLy is imposed to make sure that PP holds only between fillers (trivially, $PPxy \rightarrow FLx \;\&\; FLy$), so as to avoid that a filler would be a part of a slot.

We then define parthood on the basis of proper parthood in a similar way as in [5], among fillers:

(DEF2) **Filler-parthood** $Pxy :=_{def} FLx \;\&\; FLy \;\&\; [PPxy \lor (x=y)]$

We will call P (that holds between two fillers) "parthood", by contrast to the "slot parthood" P_S (that holds between a slot on one side, and either a slot or a filler on the other side).

We then define overlap between fillers in the usual way:

(DEF3) **Overlap** $Oxy :=_{def} \exists z\, Pzx \;\&\; Pzy$

4.1.2. First axioms

In this system, only slots are filled, and slots cannot fill:

(AX1) **Only slots are filled** $Fxs \rightarrow Ss$

(AX2) **Slots cannot fill** $Fxs \rightarrow \neg Sx$

Moreover, no entity fills any of its slots (intuitively, the slots of an entity can only be filled by something "smaller" than this entity):

(AX3) **No improper parthood slots** $\neg(P_S sx \;\&\; Fxs)$

Also, there is at most one filler for a given slot:

(AX4) **Max one occupancy** $Fys \;\&\; Fzs \rightarrow y=z$

From AX2, we deduce the trivial theorem that fillers and slots are disjoint:

(TH0) **Fillers are not slots** $FLx \rightarrow \neg Sx$

From AX1 and the definition of FL, we can deduce that the domain of F is FL, and its range is S; and the domain of P_S is S (and its range is HS, by definition of HS):

(TH1) **Domain and range of F** $Fxs \rightarrow FLx \;\&\; Ss$

(TH2) **Domain and range of P_S** $P_S sx \rightarrow Ss \;\&\; HSx$

Like in Bennett's system, this system does not require that each filler occupies exactly one parthood slot. This is indeed the main motivation of the theory, to explain how IEs such as the letter 'a' or 'John Doe' can occupy several parthood slots. That is, an entity can "have a part twice over", in Bennett's slogan.

However, contrarily to Bennett's system, a slot may not be filled; and slots can have slots. Indeed, we want to be able to say that on a document, the slot 'cn[]$_0$' (for "complete name") has as slots 'fn[]$_0$' (for "first name") and 'ln[]$_0$'(for "last name"), even if 'cn[]$_0$' is not filled. Or to say that a slot 'nominal group[]$_0$' has a slot 'noun[]$_0$', even if unfilled [2]. We even accept in our ontology documents that would be mere slot structures without any filling of themselves or of their slots (for example, my homework is currently only a blank page, but has already a predefined structure and thus some slots – see 5.6).

We accept the axioms stating that P_S is a strict order relation:

(AX5) **Slot-irreflexivity** $\neg P_S ss$

(AX6) **Slot-asymmetry** $P_S ts \rightarrow \neg P_S st$

(AX7) **Slot-transitivity** $(P_S ut \;\&\; P_S ts) \rightarrow P_S us$

4.1.3. Fillling and underfilling

We will now add the relation of underfilling (noted UF): a filler underfills a slot if it fills a slot of this slot:

(DEF4) **Underfilling** $UFxs :=_{def} Ss \,\&\, \exists t\, (P_S ts \,\&\, Fxt)$

Trivially, something that underfills is a filler:

(TH3) **Only fillers underfill** $UFxs \rightarrow FLx$

On the other hand, an underfiller of a slot s is not necessarily a part (in the sense of P) of something that fills s (since the larger slot s can remain unfilled). Contrast underfilling with proper parthood, as they look axiomatically very similar: x underfills z if x fills a slot of z *and z is a slot*; whereas x is a proper part of z if x fills a slot of z *and z is a filler.* We can show that both F and UF are strict orders, but those theorems are vacuously true (since the range and the domain of those two relations are disjoint).

4.1.4. Slot inheritance

The following axiom will play a pivotal role in our theory (and be discussed extensively in 5.1): if a filler x fills a slot s, any slot of x is a slot of s, and vice versa:

(AX8) **Slots of filler are identical to slots of the filled slot** $Fxs \rightarrow (P_S tx \leftrightarrow P_S ts)$

For example, if 'John Doe' fills 'cn[]$_0$', and 'fn[]$_{JD}$' is a slot of 'John Doe', then 'fn[]$_{JD}$' is also a slot of 'cn[]$_0$'. Consequently, a slot may have a slot for two different reasons. First, it might be because of its own intrinsic structure, such as 'cn[]$_0$' having intrinsically the slots 'fn[]$_0$' and 'ln[]$_0$'. Second, it might be because of the structure of its filler, such as 'fn[]$_0$' being filled by 'Jean-Marc' and therefore having as slots '1hfn[]$_{JM}$' and '2hfn[]$_{JM}$' (for respectively the 1st half of the first name and its 2nd half), filled respectively by 'Jean' and 'Marc' (but it would not have such slots if it was filled by 'John').

Using AX8, we can show the following theorems:

- The slots of a part of an entity are slots of that entity too:

(TH4) **Slot of part inheritance** $(P_S sx \,\&\, Pxy) \rightarrow P_S sy$

<u>Proof</u>: Let's suppose that $P_S sx$ & Pxy. Since Pxy, there is a t such that $P_S ty$ and Fxt. From AX8 and Fxt, we know that for any u, ($P_S ux \rightarrow P_S ut$). Applying this to u=s, from $P_S sx$ we can deduce $P_S st$. From $P_S st$ and $P_S ty$, we deduce $P_S sy$ by AX7 (**Slot-transitivity**).

- If x is a proper part of y, then y is not a proper part of x:

(TH5) **Proper parthood asymmetry** $PPxy \rightarrow \neg PPyx$

<u>Proof</u>: Suppose that PPxy and PPyx. Then x fills a slot s of y, and y fills a slot t of x. By TH4 (**Slot of part inheritance**), t is a slot of y. Thus, y fills one of its slots, which is impossible by AX3 (**No improper parthood slots**).

- If x is a proper part of y that fills s, then x underfills s:

(TH6) **Proper part of a filler underfills the filler's slot** $PPxy \,\&\, Fys \rightarrow UFxs$

<u>Proof</u>: Suppose that Fys and PPxy. There is a slot t of y filled by x. By AX8, t is also a slot of s. Thus, x fills a slot of s. That is, x underfills s.

- An underfiller of a slot does not fill this slot (and vice-versa):

(TH7) **Underfiller does not fill** $UFxs \rightarrow \neg Fxs$

Proof: Suppose that UFxs and Fxs. Since UFxs, x fills a slot t of s. Since Fxs, then any slot of s is a slot of x, by AX8. Therefore, t is a slot of x. Thus, x fills one of its slots: absurd by AX3.

4.1.5. (Filler-)parthood as a partial order

From the above, we can deduce easily the following theorems (that follow respectively from DEF2, TH5 and TH4):

(TH8) **Parthood Reflexivity**	FLx → Pxx
(TH9) **Parthood Anti-Symmetry**	(Pxy & Pyx) → x=y
(TH10) **Parthood Transitivity**	(Pxy & Pyz) → Pxz

4.2. Graphical representation

We can represent the structure of a document (filled, unfilled, or partially filled) as in figure 1, by progressive inclusion of rectangles (for IS) and ellipses (for IF):
- An ellipse x immediately inside a box s represents Fxs.
- A box s inside (not necessarily immediately inside, because of slot-transitivity and AX8) an ellipse or a box z represents $P_S sz$.
- An ellipse x immediately inside a box inside an ellipse y represents PPxy.

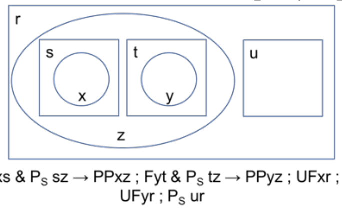

Figure 1: Various IEs and their relations

Note that contra Bennett, all boxes do not need to be filled with an ellipse (u and r are not filled, although r is underfilled) and a box can be immediately inside a box (a slot can have a slot, e.g. r has the empty slot u). We may also have e.g. x=y, since the same filler may fill several slots.

4.3. Filling axioms and theorem

Let's now add two axioms from which we can deduce new filling relations. The first one is a "descending" filling axiom: a slot of a filler is always filled:

(AX9) **Slot of a filler is filled** FLx & $P_S sx$ → ∃y Fys

We can deduce trivially from this axiom that a slot of a filler is always filled *by a proper part of the filler*:

(TH11) **Slot of a filler is filled by a proper part** FLx & $P_S sx$ → ∃y (Fys & PPyx)

For example, the slot 'fn[]$_{JD}$' of the filler 'John Doe' is filled by 'John'.

The second axiom is an "ascending" filling axiom: if all slots of a slot are filled, then this slot is also filled:

(AX10) **All sub-slots filled implies slot filled** [∀t ($P_S ts$ → ∃x Fxt)] → ∃y Fys

For example, if I fill the two only slots 'fn[]$_0$' and 'ln[]$_0$' of 'cn[]$_0$', then I have filled 'cn[]$_0$' (see 5.3 for a brief discussion).

4.4. Supplementation

We can accept an axiom of supplementation akin to the axiom found in Bennett's system. It states that if x and y are fillers that have slots, and y is not a part of x, then y has a slot that is not a slot of x, nor filled by x:

(AX11) **IF Strong Supplementation** FLx & FLy & HSx & HSy → [¬ Pyx → ∃u ($P_S$uy & ¬ ($P_S$ux ∨ Fxu)]

From AX11, we can deduce trivially:

(TH12) **IF Weak Supplementation** HSx & PPxy → ∃u ($P_S$uy & ¬ ($P_S$ux ∨ Fxu)]

Moreover, two fillers that have slots are identical iff they have the same slots:

(TH13) **IF Extensionality** (FLx & FLy & HSx & HSy) → [x=y ↔ ∀u ($P_S$ux ↔ $P_S$uy)]

Proof: Consider two fillers x and y which both have slots. If x=y, ∀u ($P_S$ux ↔ $P_S$uy). Suppose now that ∀u ($P_S$ux ↔ $P_S$uy). Suppose that x ≠ y. By anti-symmetry of P, either ¬Pyx or ¬Pxy. In the first case, by AX11, there is a u such that $P_S$uy & ¬ $P_S$ux: absurd. The second case is absurd for the same reason. Therefore, x=y.
Note that fillers with no slots can still be identical, e.g. 'a' is identical with 'a'. Moreover, contrarily to classical mereology, two fillers can have the same filler proper parts without being identical, such as '1^{st} letter$[a]_1$ 2^{nd} letter$[b]_1$' and '1^{st} letter$[b]_2$ 2^{nd} letter$[a]_2$'.

4.5. Comparison with Bennett's mereology

In the following, for an integer *n*, "A*n*" or "T*n*" (e.g. "A3") refer to an axiom or theorem in Bennett's system [4], whereas "AX*n*" or "TH*n*" (e.g. "AX3") refer to an axiom or theorem in our theory. There are several important changes in the system presented above, compared to Bennett's mereology. First, not all slots are "slots of something": there can be free-floating slots. Second, a filler does not have an "improper slot" that it itself fills (cf. AX3, contra A4): intuitively, a slot is "smaller" than the thing it is a slot of. Third, there can be empty slots, and therefore, contra Bennett's A7 that states that each slot has a single occupant, we merely accept a weaker axiom of maximum one occupant (AX4). Fourth, slots can have slots, contra A3 – and therefore, we introduced the notion of underfilling in DEF4.

Bennett deduces from her axioms that slot parthood is a partial order, but since slots do not have slots in her system, those order properties are vacuous. In our system, those order properties are quite substantial (and necessary for many of our demonstrations), and we accepted them as axioms. Bennett also accepts as an axiom the Slot of Part Inheritance (A5). In our case, we accepted the more general axiom AX8 (which would have no sense in Bennett's system, in which slots do not have slots); we then used it to prove Slot of Part Inheritance as a theorem (TH4).

We accepted a strong supplementation theorem on fillers (AX11) similar to Bennett's axiom A8 (what Bennett calls "Slot Strong Supplementation" is what we call "IF Strong Supplementation", for reasons that will become clear in 5.1.1). Note that the last part of Bennett's A8 axiom reads "∃u ($P_S$uy & ¬ $P_S$ux)", whereas it reads for us "∃u ($P_S$uy & ¬ ($P_S$ux ∨ Fxu)]": indeed, Bennett's accepts improper parthood slot, but we don't, so we also want to make sure that this supplementary slot u is not filled by x.

Associate theorems of weak supplementation and extensionality trivially follow (TH12, TH13), like in Bennett's system (T13, T14).

5. Discussion

We will discuss here several ways to complete the system proposed above.

5.1. The identification of the slots of a filled slot and the slots of its filler

We have proposed above a relatively simple theory of ISs and IFs, that identifies by AX8 the slots of a filler with the slots of the slot it fills. However, there are challenges of two kinds if we want to extend this theory with more sophisticated considerations.

5.1.1. Challenges

The first challenge is that the theory above would not fit well with a theory of diachronic evolution of the slot structure of a slot (but see the footnote in 5.6 that briefly discusses whether we want to have a diachronic theory of IEs in a first place). Suppose that the slot '$cn[]_0$' has the slots '$fn[]_0$' and '$ln[]_0$', and that the filler 'John Doe' has the slots '$fn[]_{JD}$' and '$ln[]_{JD}$'. As soon as 'John Doe' fills '$cn[]_0$', the slots '$fn[]_{JD}$' and '$ln[]_{JD}$' appear in '$cn[]_0$'. Such change is somewhat odd.

The second challenge pertains to supplementation. We have here accepted an axiom of strong supplementation among fillers (AX11) similar to Bennett's. However, because our system accepts slots of slot, one might want to introduce axioms of supplementation purely at the level of slots, without any need of mediation by fillers. For this, we could first define a notion of overlap between slots as follows: s and t slot-overlap just in case they share a parthood slot u, one is a parthood slot of the other, or they are identical (this latter mention is important, since a slot that does not have any slot does not share a slot with itself, but we want to state that any slot overlaps itself):

(DEF5) **Slot-overlap** $O_Sst :=_{def} Ss \,\&\, St\ [\exists u\,(P_Sus \,\&\, P_Sut)] \vee P_Sst \vee P_Sts \vee (s=t)]$

This would enable to formulate an axiom of strong supplementation among slots:

(AX12) **IS Strong Supplementation** $Ss \,\&\, St \,\&\, HSt \rightarrow [(\neg P_Sts \,\&\, s \neq t) \rightarrow (\exists u\, P_Sut \,\&\, \neg O_sus)]$

From this axiom, we could then deduce easily corresponding theorems of weak supplementation and extensionality:

(TH13) **IS Weak Supplementation** $St \,\&\, P_Sst \rightarrow \exists u\,(P_Sut \,\&\, \neg O_sus)$

(TH14) **IS Extensionality** $Ss \,\&\, St \,\&\, HSs \,\&\, HSt \rightarrow [s=t \leftrightarrow \forall u\,(P_Sus \leftrightarrow P_Sut)]$

Ideally, we would use AX12 (**IS strong supplementation**) to prove **IF strong supplementation** as a theorem, instead of accepting it as an axiom as we did here. However, **IS extensionality** is not compatible with our AX8. Indeed, if two slots are filled by the same filler that has slots, they have exactly the same slots by AX8; and by **IS Extensionality** (TH14), they would be identical. This would defeat the goal of this system that aims at enabling an IE to have a part twice (or more) over. To avoid this conclusion, one could relax AX8, for example by introducing the notion of "twin-slot".

5.1.2. Twin-slots

AX8 is equivalent to the conjunction of the two following axioms:

(AX8.1) **Slots of a filled slot are slots of the filler** $Fxs \ \& \ Psts \rightarrow Pstx$

(AX8.2) **Slot of a filler are slots of the slot it fills** $Fxs \ \& \ Pstx \rightarrow Psts$

To relax AX8, one could relax AX8.1, AX8.2, or both. However, if one (or both) of those axioms is abandoned, we may want to replace them by weaker axioms. For example, it seems sensible that because 'cn[]$_0$' has two slots 'fn[]$_0$' and 'ln[]$_0$', a filler x of 'cn[]$_0$' should have two slots 'fn[]$_x$' and 'ln[]$_x$'. To account for this and relax AX8.1, one could introduce the notion of "twin-slot" of a slot's slot. A minimal requirement for such twin-slot would be that if x fills t and s_t is a slot of t, then there is a slot s_x of x, called "twin-slot of s_t in x", such that s_x is filled by a whenever s_t is filled by a:

(AX8.1') **Twin-slot in a filler** $Fxt \ \& \ Ps s_t t \rightarrow \exists s_x, Ps s_x x \ \& \ \forall a(Fas_t \leftrightarrow Fas_x)$

Using AX8.1', TH11 and AX4, we can show that a slot of t and its twin slot in x are filled by the same proper part of x:

(TH15) **Filling of a twin-slot in a filler** $Fxt \ \& \ Ps s_t t \rightarrow$
$\exists s_x, \exists! y \ Ps s_x x \ \& \ Fys_t \ \& \ Fys_x \ \& \ PPyx$

For example, if 'John Doe' fills 'cn[]$_0$', it has two slots 'fn[]$_{JD}$' and 'ln[]$_{JD}$' that are twin-slots of respectively 'fn[]$_0$' and 'ln[]$_0$'. Both 'ln[]$_{JD}$' and 'ln[]$_0$' are filled by 'Doe'; but if 'cn[]$_0$' was instead filled by 'Jane Smith', then 'ln[]$_0$' would be filled by 'Smith', whereas 'ln[]$_{JD}$' would still be filled by 'Doe' (and it would then not be a twin slot of 'ln[]$_0$').

Similarly, one might also want to introduce the notion of twin-slot of a filler's slot. Future work should determine which of those axioms, and/or others, should be endorsed.

5.2. Slot levels

We may want to define a hierarchy of sublevels among slots. For example, a diagnostic report that would occupy a slot s_0 could have the slots 'patient[]$_0$ condition[]$_0$ doctor[]$_0$', where the slot 'patient[]$_0$' is composed by the slots 'patient_fn[]$_0$' and 'patient_ln[]$_0$'. Note that according to slot-transitivity, 'patient_fn[]$_0$' and 'patient_ln[]$_0$' are also slots of s_0. But we might want to state that the slots 'patient[]$_0$', 'condition[]$_0$' and 'doctor[]$_0$' are first-sublevel slots of s_0, whereas the slots 'patient_fn[]$_0$' and 'patient_ln[]$_0$' are second-sublevel slots of s_0. We can define the various sublevels as follows:

- A 1st-level slot of s is a slot of s that is not a slot of a slot of s.
- A 2nd-level slot of s is a slot of a slot of s that is not a slot of a slot of a slot of s.

etc.

Note that in our axiomatization, there is no axiom that forces the existence of 1st-level, 2nd-level etc. slots. That is, there is no discreteness axiom that rules out the existence of a dense set of sublevels. Such axioms might be added in future work.

5.3. Mereological sum

We may want to formalize mereological sum of several fillers of several slots. For example, elaborating on AX10, we may want to state that if all slots of a slot s are filled, the filler that fills s is the mereological sum of those fillers.

More innovatively, we may want to introduce an entity that would be composed by a filler and the slot it fills, such as the slot 'patient[]$_0$' and its filler 'John Doe'. This would indeed have consequences for aboutness (see subsection 5.4). A possible way might be to represent this as a mereological sum of the filler and its slot. Such an account could be compared to Koslicki's theory of the composition of material objects [20], which holds that objects have two proper parts: material parts and formal parts. As her analysis goes, for instance, Michelangelo's statue David is composed of an amount of marble (material part) and, say, the "David-wise structure" (formal part). Indeed, Koslicki states that a general notion of structure can be characterized as an entity that offers available "*positions* or *places*" (recall "ISE" and "slot" in our discussion). A mereological sum of a filler and its slot that would be completed by some kind of arrangement among subslots would thus seem to be very much in Koslicki's spirit.

5.4. Aboutness

We will not propose a formal theory of aboutness, but only give a few pointers of how we can extend IAO's theory of aboutness with an aboutness of ISs. We will consider here that an informational entity can be about several kinds of entities, such as a particular, a class, or a state of affair [13]. This is relatively classical at the level of fillers. For example, in $diag_0$, 'John Doe' is about the particular human John Doe and 'Flu' is about the class *Flu*. However, we suggest that slots can also be about a variety of things. For example, 'patient$[]_0$' would be about the class *Patient* and 'condition$[]_0$' would be about the class *Medical condition*. Then, if we accept the mereological sum of a slot and its filler as explained earlier, the sum of 'John Doe' and the slot it occupies 'patient$[]_0$' might be about the relation of instantiation of the class *Patient* by John Doe. Note however that all slots are not about something – consider e.g. the slot '2nd letter$[]_0$'. Such considerations should be integrated into a full theory of aboutness of ICE – something that still needs to be developed in IAO.

5.5. Refusing supplementation

There might be reasons to refuse TH12 - **IF Weak Supplementation** (and therefore AX11 - **IF Strong Supplementation**) in our mereological system. Indeed, we may want to accept to have PPxy while y having no slots that are not slots of x, except the one filled by x. For example, suppose that Mr. J's last name is composed by only one letter, 'J'. He needs to fill some administrative form that has the slot 'last name[]'. He fills it with 'J'. The slot 'last name[]' is filled with this filler 'J'_{Name}, that refers to Mr. J. And this filler has one unique slot 'first letter[]', that is filled with the filler $\text{'J'}_{\text{Letter}}$, which is a letter that refers to nothing. That is, Mr. J fills two slots by drawing the same sign.

However, following a suggestion by Masolo and Vieu [2], one might instead introduce a relation of composition between a word and the chain of character it is made of, such that this relation would not be identical to parthood. In such a case, 'J'_{Name} would be *constituted* by the character $\text{'J'}_{\text{Letter}}$, but would not *have it as part*, and thus it would not constitute a counter-example to supplementation axioms.

5.6. The diachronic identity of documents and creation of ISs

The diachronic identity of documents is a topic that has been little studied. To illustrate its complexity, suppose that I start working on a homework. In front of me, I have a blank sheet of paper. At t_1, I decide that this paper will be the physical carrier of my homework: I decide that I will write my name on the top left, the date on the top right, the body of my text on the paper, divided in three parts. At t_2, I have written my name on the top left. At t_3, I have written the date on the top right. At t_4, I have written the three parts of the body of my text, and finished the homework.

It would be desirable to have a theory of identity according to which it is the same homework that evolves while I am filling it; that is, that there exists a unique homework IE at t_2, t_3 and t_4 - and maybe even already at t_1. The theory we developed earlier is compatible with such diachronic identity considerations: the homework remains the same document from t_2 (and maybe even already from t_1) to t_4, although some new parts (new IFs) appear[3].

Interestingly, the relevant ISs have arguably already appeared at t_1. Of course, we do not claim that something physically changed in the composition of the sheet of paper when I made this decision at t_1 to write my homework on this sheet of paper. The fact that it changed while not changing in physical structure (that is, that it undergoes a Cambridge change [22]) only emphasizes the cognitive and social nature of ISs (something that is also true for IFs, as they depend on the existence of a semiotic system): it is my cognitive act (maybe mirrored in other cognitive agents) to structure my document with a name, a date, and a body of work in three parts that created those ISs.

Similarly, a database, a patient chart, a distribution report can remain identical while having a changing content. Note that the number and nature of the ISs of an entity can change in two ways. First, trivially, IFs will themselves have ISs, which will, by slot inheritance, be ISs of the overall documents; therefore, new slots appear (consider again the “Jean-Marc” example in section 4.1.4). Second, the ISs of a document can change depending on how agents change another part of the document. Consider for example a response sheet to a poll, where the positive answer to “Are you a national of another country than Canada?” will bring the IS to be filled by the nationality of the respondent.

6. Conclusion

We have thus introduced an axiomatic system for the mereology of informational entities, using the notions of information slots and information fillers. Inspired by Bennett, this system is different in several important respects, in particular in having free-floating slots, slots of slots and empty slots.

Important extensions will include relaxing AX8, introducing Slot supplementation axioms and mereological sums, as well as analyzing the aboutness of slots and fillers. Also, one should analyze the various forms in which IFs and ISs can be concretized, in particular when a message is conveyed as a process (e.g. spoken language or morse code). This could strengthen the basis of ontologies of documents such as IAO, and application ontologies founded on it, such as the Prescription of Drugs Ontology (PDRO) [23], the LABoratory Ontology (LABO) [24], the ontology of document acts [25], or the Informed Consent Ontology (ICO) [26].

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[3] Note that this raises an interesting difficulty: if I make two copies of the homework at an earlier stage of development and then both evolve in different ways, it seems that they would remain identical despite evolving differently, which is counter-intuitive. This kind of problem has been studied by Parfit in the case of personal identity [21]. One possible solution, in a Parfitian spirit, would then be to drop the notion of diachronic identity for informational entities.

References

[1] A. Barton, P. Fabry, J.-F. Ethier, A classification of instructions in drug prescriptions and pharmacist documents, in: *Proceedings of the 10th International Conference on Biomedical Ontology (ICBO 2019)*, Buffalo, New York, USA, accepté, pp. 1–7.

[2] C. Masolo, L. Vieu, Graph-Based Approaches to Structural Universals and Complex States of Affairs, in: *Formal Ontology in Information Systems: Proceedings of the 10th International Conference (FOIS 2018)*, IOS Press, 2018, p. 69.

[3] R. Arp, B. Smith, A.D. Spear, *Building Ontologies with Basic Formal Ontology*, The MIT Press, 2015.

[4] K. Bennett, Having a part twice over, *Australasian Journal of Philosophy* **91** (2013), 83–103.

[5] P. Simons, *Parts: A study in ontology*, Clarendon Press, Oxford, 1987.

[6] R. Casati, A.C. Varzi, *Parts and places: The structures of spatial representation*, Mit Press, Cambridge, MA, 1999.

[7] C. Masolo, S. Borgo, A. Gangemi, N. Guarino, A. Oltramari, *The WonderWeb library of foundational ontologies and the DOLCE ontology. WonderWeb (EU IST project 2001-33052) deliverable D18*, LOA-ISTC-CNR, 2003.

[8] A.J. Cotnoir, Strange parts: The metaphysics of non-classical mereologies, *Philosophy Compass* **8** (2013), 834–845.

[9] N. Effingham, J. Robson, A mereological challenge to endurantism, *Australasian Journal of Philosophy* **85** (2007), 633–640.

[10] F. Cancedda, Functional requirements for authority data: a conceptual model: Final report, December 2008, *Bollettino AIB (1992-2011)* **50** (2010), 131–132.

[11] S. Peroni, M. Palmirani, F. Vitali, UNDO: the United Nations system document ontology, in: *International Semantic Web Conference*, Springer, 2017, pp. 175–183.

[12] L. Bountouri, M. Gergatsoulis, The semantic mapping of archival metadata to the CIDOC CRM ontology, *Journal of Archival Organization* **9** (2011), 174–207.

[13] B. Smith, W. Ceusters, Aboutness: Towards foundations for the information artifact ontology, in: *Proceedings of the 6th International Conference on Biomedical Ontology*, CEUR Workshop Proceedings, Lisbon, Portugal, 2015, pp. 1–5.

[14] R. Rudnicki, *An Overview of the Common Core Ontologies*, CUBRC, Inc., Buffalo, 2019.

[15] C. Masolo, L. Vieu, E. Bottazzi, C. Catenacci, R. Ferrario, A. Gangemi, et al., Social Roles and their Descriptions., in: D. Dubois, C. Welty (Eds.), *Proceedings of the 9th Int. Conf. on Principles of Knowledge Representation and Reasoning (KR 2004)*, AAAI Press, Menlo Park, CA, 2004, pp. 267–277.

[16] N. Arora, R. Westenthaler, W. Behrendt, A. Gangemi, Information object design pattern for modeling domain specific knowledge, in: *1st ECOOP Workshop on Domain-Specific Program Development*, 2006.

[17] P. Hawke, Theories of aboutness, *Australasian Journal of Philosophy* **96** (2018), 697–723.

[18] J.A. Bateman, Ontology, Language, Meaning: Semiotic Steps Beyond the Information Artifact., in: *Ontology Makes Sense: Essays in Honor of Nicola Guarino*, IOS Press, Amsterdam, 2019, pp. 119–135.

[19] A. Galton, On generically dependent entities, *Applied Ontology* **9** (2014), 129–153.

[20] K. Koslicki, *The structure of objects*, Oxford University Press on Demand, 2008.

[21] D. Parfit, *Reasons and persons*, OUP Oxford, 1984.

[22] C. Mortensen, Change and Inconsistency, in: E.N. Zalta (Ed.), *The Stanford Encyclopedia of Philosophy*, Spring 2020, Metaphysics Research Lab, Stanford University, 2020.

[23] J.-F. Ethier, A. Barton, R. Taseen, An ontological analysis of drug prescriptions, *Applied Ontology* **13** (2018), 273–294.

[24] A. Barton, P. Fabry, L. Lavoie, J.-F. Ethier, LABO: An ontology for laboratory test prescription and reporting, in: *Proceedings of the Joint Ontology Workshops 2019 (JOWO 2019)*, Graz, Autriche, 2019, pp. 1–9.

[25] M. Brochhausen, M.B. Almeida, L. Slaughter, Towards a formal representation of document acts and resulting legal entities, in: *Johanssonian Investigations: Essays in Honour of Ingvar Johansson on His Seventieth Birthday*, Walter de Gruyter, 2013.

[26] Y. Lin, M.R. Harris, F.J. Manion, E. Eisenhauer, B. Zhao, W. Shi, et al., Development of a BFO-based informed consent ontology (ICO), *Bioinformatics* **1327** (2014), 84–86.